# Raising Sfermion Masses by Adding Extra Matter Fields

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(October 2003)

## Abstract

The renormalization group flow of soft supersymmetry breaking masses is sensitive to the field contents of the theory one considers. We point out that the addition of extra vector-like matter fields to the minimal supersymmetric standard model raises the masses of squarks and sleptons relative to those of gauginos. We discuss its phenomenological implications. Besides an obvious effect to the superparticle mass spectrum, we find that radiative corrections from heavier stop loops increase the lightest CP-even Higgs boson mass. We also discuss impact on models with no-scale boundary conditions. It turns out that, unlike the minimal case, staus can become heavier than a B-ino like neutralino, which is cosmologically favored.

It has widely been believed that low energy supersymmetry (SUSY) [1] is the most promising approach to solve the naturalness problem on the electroweak scale inherent in the standard model of particle physics. If this line of reasoning is correct, one of the most interesting tasks is to reveal the nature of the mechanism of supersymmetry breaking and its mediation to the standard model sector. Superparticle masses which are evaluated at the electroweak scale are supposed to be given at high energy scale in a hypothetical fundamental theory. A key ingredient to connect the quantities at the different scales is renormalization group (RG) evolution.

It is known that the RG flow depends on the field contents of the theory one considers. In this paper, we will examine how the addition of extra vector-like matter fields to the minimal supersymmetric standard model (MSSM) affects the RG evolution and superparticle mass spectrum. Though the MSSM particle contents successfully explain the unification of three gauge coupling constants, there is still some room to add extra vector-like matter fields with the restriction that they should constitute full $SU(5)$ multiplets in order not to destroy the success at least at one loop level. Introduction of extra matter fields is often considered to solve some difficulty of particle physics models. Examples include a hadronic axion model [2], an attractive solution to the strong CP problem.

The key observation we make in this paper is that, as we will discuss shortly, the change of the RG evolution of the gauge coupling constants as well as the gaugino masses due to the presence of the extra matter fields raises the sfermion masses when compared to the gaugino masses at the electroweak scale. This observation has a lot of phenomenological implications. Besides an obvious remark on the modification of the superparticle mass spectrum which will hopefully be measured in future collider experiments [3], we will point out that the larger stop mass implied by the heavier sfermion mass spectrum makes the Higgs boson mass large due to radiation corrections from top-stop loop [4]. In fact, the experimental Higgs mass bound gives a rather severe constraint on models where scalar masses given at high energy scale are small. The RG effect with the extra matter fields will somewhat relax the constraint.

We will also discuss implications to models with no-scale boundary conditions. In this scenario, the scalar masses vanish at the boundary, and thus it can be a natural solution to the supersymmetric flavor problem. This type of boundary conditions was realized originally in the no-scale model [5] and also in the context of heterotic string theory [6]. It was recognized [7] that the vanishing scalar mass as well as a vanishing $A$-parameter is a common feature of the models where the hidden and observable sectors are appropriately separated in the Kähler potential, and then the gaugino masses can be the only source of the SUSY breaking masses. It is interesting to note that the splitting may naturally be realized in the geometrical setting where the hidden-sector brane is sequestered from the standard-model brane [8,9]. In this setup, the gauginos can acquire SUSY breaking masses if they propagate in the bulk and couple to the SUSY breaking fields on the hidden brane [10]. Despite this attractive feature, the minimal setup with the RG evolution starting from the grand-unified-theory (GUT) scale faces a serious phenomenological difficulty. The point is a coincidental degeneracy in masses of right-handed sleptons and B-ino. A previous study showed that the cosmological requirement that the neutralino to be the lightest superparticle (LSP) (not charged slepton) gives very stringent constraint on the upperbound of the superparticle mass spectrum [7,11,12]. Moreover with this constraint, the predicted Higgs

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mass would be lower than its experimental bound, and thus this interesting idea would conflict with experiments. There have been proposed several mechanisms to avoid this problem in the literature, including possible RG flow above the GUT scale [13–15], light gravitino or axino LSP scenario, and also non-universal gaugino masses [16]. Here we will propose an alternative solution with the addition of the extra matter fields.

We begin by discussing the RG evolution of soft SUSY breaking mass parameters. The RG equations (RGEs) for the gauge coupling constants $\alpha_i$ in $N = 1$ supersymmetry are written

$$\mu\frac{d\alpha}{d\mu} = -\frac{b}{2\pi}\alpha^2 \ , \tag{1}$$

$$b = C_2(G) - \sum_{\text{chiral}} T(R) \tag{2}$$

where $C_2(G) = N$ for $G = SU(N)$ and $T(R)$ is defined as $\text{Tr}T^aT^b = T(R)\delta^{ab}$ with $T(\text{fund}) = 1/2$ and in the second term of (2) summation over the chiral multiplets is understood. When one considers MSSM particle contents with additional extra vector-like matter fields, the above become

$$\mu\frac{d\alpha_i}{d\mu} = -\frac{\beta_i - N_{\text{ex}}}{2\pi}\alpha_i^2 \ , \tag{3}$$

where $i$ runs from 1 to 3, with $\beta_i = (3, -1, -33/5)$ for $SU(3)_C \times SU(2)_L \times U(1)_Y$. Here we have assumed that the extra matter multiplets consist of full multiplets in terms of $SU(5)$ GUT, which is requisite not to destroy the successful gauge coupling unification in the MSSM, and we denote the number of the extra vector-like matter multiplets by $N_{\text{ex}}$, whose normalization is given such that $N_{\text{ex}} = 1$ for one pair of $\mathbf{5}$ and $\bar{\mathbf{5}}$ representations and $N_{\text{ex}} = 3$ for one pair of $\mathbf{10}$ and $\overline{\mathbf{10}}$ representations.

Similarly the RGEs for the gaugino masses are given as

$$\mu\frac{dM_i}{d\mu} = -\frac{\beta_i - N_{\text{ex}}}{2\pi}\alpha_i M_i \ . \tag{4}$$

Throughout this paper, we consider the case where the extra matter fields do not have large Yukawa coupling to the ordinary quarks and leptons and to the Higgs fields. In this case the RG evolution of the SUSY breaking scalar masses is not modified by the extra matter fields at one-loop level.

For simplicity, we assume that all extra matter fields have a common mass, $M_{\text{ex}}$. We also assume that they do not mediate any non-trivial SUSY breaking (unlike gauge mediation) and that the threshold effects to soft masses, when they decouple, are negligibly small, which is justified when the soft SUSY breaking $B$-parameters for the extra matter fields are not much larger than the gaugino masses and the number of the extra matter fields is not extremely large. The latter condition is always fulfilled in our case, because it is restricted by the perturbativity of the gauge coupling constants. With this setup, we simply solve the RGEs with the extra multiplets above $M_{\text{ex}}$, and below this scale use is made of the RGEs of the MSSM, with the trivial matching condition without any threshold corrections imposed at $M_{\text{ex}}$.

It is then straightforward to solve the RGEs of this system. Fig. 1 demonstrate how the RG evolution changes in the presence of the extra matter fields. Here the gaugino masses at the electroweak scale are taken to be the same between the two specific cases of $N_{\text{ex}}$ ($N_{\text{ex}} = 0$ for Fig. 1(a) and $N_{\text{ex}} = 3$ for (b)). To emphasize the effects of the RG evolution, we assume that the soft scalar masses vanish at the boundary of the RG evolution, which is assumed to be the GUT scale. Also the decoupling scale of the extra matter fields is chosen, somewhat arbitrarily, to $M_{\text{ex}} = 10^4$ GeV and $\tan\beta$, which is the ratio of the two Higgs vacuum expectation values, is taken $\tan\beta = 10$. We also assume that the gaugino masses have a common origin, that is, universal gaugino mass. First, we find that the gaugino masses at the low-energy are suppressed in the existence of the extra matter fields, which is easily understood by noticing that the gauge couplings are less asymptotic free at ultra-violet (UV) region. Put another way, in order to obtain the same gaugino masses at the low-energy scale, one has to start with a larger gaugino mass at the high-energy scale. Combined with the fact that the gauge coupling constants are also large in the UV side, the scalar fields acquire their soft masses at the UV scale, which thus significantly enhance the ratio of the sfermion masses with respect to the gaugino masses when compared with the case of no extra matter.

Here it is instructive to give analytic formulae for the soft masses. For instance the $SU(2)_L$ gaugino mass and the soft scalar masses of the first two generations are solved, when we impose a universal soft scalar mass, $m_0$, and a universal gaugino mass, $M_{1/2}$ at the GUT scale $\sim 2 \times 10^{16}$ GeV, as:

$$M_2(\text{EW}) \simeq 0.34 M_{1/2}, \tag{5}$$
$$m_{\tilde{q}}^2(\text{EW}) \simeq m_0^2 + 2.6 M_{1/2}^2 \simeq m_0^2 + 22 M_2^2(\text{EW}), \tag{6}$$
$$m_{\tilde{\ell}_L}^2(\text{EW}) \simeq m_0^2 + 0.35 M_{1/2}^2 \simeq m_0^2 + 3.0 M_2^2(\text{EW}), \tag{7}$$
$$m_{\tilde{\ell}_R}^2(\text{EW}) \simeq m_0^2 + 0.12 M_{1/2}^2 \simeq m_0^2 + 1.0 M_2^2(\text{EW}). \tag{8}$$

where we take $N_{\text{ex}} = 3$ and $M_{\text{ex}} = 10^4$ GeV. The argument "EW" represents that they are quantities evaluated at the electroweak scale. In practice, we have set the renormalization scale at 500 GeV. These formulae should be compared with those of the MSSM case:

$$M_2(\text{EW}) \simeq 0.84 M_{1/2}, \tag{9}$$
$$m_{\tilde{q}}^2(\text{EW}) \simeq m_0^2 + 4.9 M_{1/2}^2 \simeq m_0^2 + 6.9 M_2^2(\text{EW}), \tag{10}$$
$$m_{\tilde{\ell}_L}^2(\text{EW}) \simeq m_0^2 + 0.49 M_{1/2}^2 \simeq m_0^2 + 0.69 M_2^2(\text{EW}), \tag{11}$$
$$m_{\tilde{\ell}_R}^2(\text{EW}) \simeq m_0^2 + 0.15 M_{1/2}^2 \simeq m_0^2 + 0.21 M_2^2(\text{EW}). \tag{12}$$

The enhancement of the scalar masses relative to the gaugino masses at the electroweak scale is apparent. In fact what happens here is that the gaugino masses at low energy become smaller while the sfermion masses do not change so much when $M_{1/2}$ is fixed, making the scalar/gaugino mass ratio larger.

We should note here that the contributions from the extra matter fields become less significant for smaller $N_{\text{ex}}$ and for higher $M_{\text{ex}}$. When $M_{\text{ex}}$ is larger than $10^{10}$ GeV, the effect becomes negligible.

Having established the increase of the sfermion masses, we now consider phenomenological implications of the presence of the extra matter multiplets. An immediate consequence is the modification of the superparticle mass spectrum. This will be particularly important in the future program to determine the mediation mechanism of the supersymmetry breaking by tracing the RG flow to higher scale with the superparticle masses which, we hope, will be measured at future collider experiments as input parameters. In this process, one has to keep in mind that the presence of the extra matter fields can drastically change the RG flow from that of the MSSM. We note that the effect of the extra matter fields cannot be absorbed by the lift of the universal scalar mass $m_0$, rather it gives a richer structure of the superparticle mass spectrum.

In SUSY models, the experimental bound on the lightest CP-even Higgs boson mass is known to provide a rather severe constraint on the soft mass parameters, that is, larger soft masses are favored to enhance the Higgs mass. In fact, the Higgs mass bound from LEP II experiment cannot be satisfied at tree level and radiative corrections play an important role. The radiative corrections mainly depend on the stop masses and a larger stop mass yields a heavier Higgs mass [4]. Thus the addition of the extra matter fields can significantly relax the constraint on the parameter space from the Higgs boson mass bound. We will give an explicit example shortly.

Another implication we would like to discuss is on the so-called no-scale scenario. In this scenario, the soft masses satisfy the following no-scale boundary conditions:

- vanishing soft scalar masses: $m_0 = 0$,

- vanishing trilinear scalar couplings: $A = 0$,

- (generally) non-vanishing Higgs mixing parameter: $B$,

- non-vanishing gaugino masses: $M_{1/2}$.

These are given at some fundamental scale, which we assume to be the GUT scale.

With the MSSM matter contents ($N_{\rm ex} = 0$), the right-handed slepton obtains a mass of $m^2_{\tilde{\ell}_R}({\rm EW}) \approx 0.84 M^2_1({\rm EW})$, and thus it is smaller than $M_1({\rm EW})$. Therefore the B-ino like neutralino can be lighter than the right-handed slepton only when there is substantial mixing in the neutralino mass matrix, which is the case when the gaugino mass is not larger than the $Z$-mass scale. In fact, a severe upperbound on the neutralino mass is obtained from the requirement that it becomes the LSP, as was shown in [7,11,12].

In Fig. 2, the masses of the lightest neutralino and the lighter stau at the electroweak scale are shown. Here we take $N_{\rm ex} = 0 - 4$, $\tan\beta = 10$ and $M_{\rm ex} = 10^4$ GeV. We also take the gaugino mass at the GUT scale in the range $M_{1/2} = 100 - 1500$ GeV. The comparison of the masses of the two superparticles yields the region allowed by the cosmological argument that the stable LSP should be neutral. The shadow region is excluded, as the charged stau is the LSP. One readily finds that in the MSSM case ($N_{\rm ex} = 0$), the allowed region is very restricted, where the upperbound of the neutralino mass is about 70 GeV. The region becomes somewhat enhanced for lower $\tan\beta$ (*e.g.* the upperbound becomes 110 GeV for $\tan\beta = 3$), but still the allowed region is quite limited. On the other hand, as the number of the extra matter increases, the allowed region where the lightest neutralino (which is B-ino-like) becomes the LSP becomes drastically larger. In fact, one finds that for $N_{\rm ex} \gtrsim 2$ the lightest neutralino always becomes the LSP.

The contour of the Higgs mass of $m_h = 115$ GeV, which roughly corresponds to the present experimental lower bound [17], is also drawn in the same figure. Here we have used `FeynHiggsFast` package [18] to compute the Higgs mass. We can see that the constraint from the Higgs mass becomes relaxed significantly as $N_{\mathrm{ex}}$ increases. In fact, in the MSSM case (i.e. with no extra matters), the cosmologically allowed region does not satisfy the Higgs mass bound, and thus the whole region is excluded.[1] However in the presence of the extra matter fields, the sfermions, especially the stop, become heavier compared to the gaugino masses, and thus the Higgs mass bound gives less restrictive constraint on the gaugino-like neutralino mass $m_{\chi_1^0}$.

In Table I, comparison of various quantities is made for $N_{\mathrm{ex}} = 0 - 4$ when $M_1$ at the electroweak scale is fixed to be 100 GeV. Recall that the scalar masses increase as $N_{\mathrm{ex}}$ increases. Thus the SUSY contributions to low energy observables reduce for larger $N_{\mathrm{ex}}$. In fact the branching ratio of $b \to s\gamma$ gradually approaches to the value of SM prediction as $N_{\mathrm{ex}}$ becomes larger. Also the SUSY contribution to the muon $g - 2$ is decreased. On the other hand, the Higgs mass becomes larger by the the enhancement of the radiative corrections and the constraint from the mass is relaxed, as was already mentioned.

Some of the features discussed above are quantitatively modified when $\tan\beta$ is large. A crucial difference comes from the fact that the Yukawa coupling of the tau lepton is enhanced by $\tan\beta$, and becomes significantly large when $\tan\beta$ is large. The large Yukawa coupling reduces the stau mass at low energy scale through the RG flow, and hence the requirement that the LSP should be neutral gives a stronger constraint on the parameter space. We explicitly checked the case of $\tan\beta = 30$. We found that the stau mass is reduced by about 100 GeV for $\tan\beta = 30$ while the neutralino mass is almost unchanged. As a result, $N_{\mathrm{ex}} = 0, 1$ are completely excluded by cosmological argument. For $N_{\mathrm{ex}} = 2$, only the region where the lightest neutralino mass is heavier than 300 GeV is allowed. The constraint is somewhat relaxed for $N_{\mathrm{ex}} = 3$, with the neutralino mass required to be larger than 100 GeV. Almost all regions are allowed for $N_{\mathrm{ex}} = 4$. At the same time, the contour lines of the Higgs mass are also lowered about 100 GeV on the stau v.s. the neutralino mass line.

In the above analysis, we have implicitly taken the top mass $m_t$ to be 175 GeV. We also analyzed the case of $m_t = 180$ GeV. We found that the lines of the stau-neutralino masses are almost intact because the effects of the top Yukawa couplings come through the determination of the supersymmetric higgsino mass parameter, $\mu$, and in our case $\mu$ is large and thus its effects are decoupled. On the other hand, the Higgs mass changes significantly since the radiative corrections are proportional to the 4-th power of the top mass. In fact the computed Higgs mass is found to increase by about 2 or 3 GeV.

Finally we would like to make a brief comment on how the recent WMAP result [19,20] on the abundance of the dark matter, $\Omega_{DM} h^2 \approx 0.11$, affects on our scenario. For the bino-like LSP, its relic abundance calculated under the standard thermal history of the Universe tends to be larger than the WMAP result. One way to evade this difficulty is invoke efficient coannihilation [21,22] with the sleptons. It requires that the stau mass is quite degenerate with the neutralino mass. In our case, this is achieved by appropriately adjusting the mass $M_{\mathrm{ex}}$ to make the effect of the extra matter fields less significant. Another possibility is to

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[1]This is also the case for lower $\tan\beta$. We checked this explicitly for $\tan\beta = 3$.

assume non-standard thermal history of the Universe below the weak scale, such as late-time entropy production to dilute the abundance of the neutralinos. Note that when the LSP were stau, the entropy production would not be able to reduce its relic density enough to survive the severe constraint from charged massive stable particle searches [23]. We leave further study on the issue of the relic abundance of the neutralino LSP for future publication.

To summarize, we have pointed out that the addition of the extra vector-like chiral multiplets can significantly change the RG evolution of the soft SUSY breaking masses in the MSSM. In particular, we found that the sfermion masses are enhanced relative to the gaugino masses at low energy. We also illustrated phenomenological implications of this effect, such as the change of the superparticle mass spectrum, the enhancement of the lightest CP-even Higgs mass and also some impact on the no-scale scenario.

## ACKNOWLEDGMENT

## NOTE ADDED

After submitting the paper, we learnt that the addition of the extra matter fields was also considered in Ref. [24] in a different context, *i.e.* in the comparison with higher order effects to the RG evolution. We thank D.R.T. Jones for drawing our attention to this paper.

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TABLES

| $N_{\rm ex}$ | $Br(b \to s\gamma)$ ($\times 10^{-4}$) | $a_\mu = (g_\mu - 2)/2$ ($\times 10^{-9}$) | $m_h^0$ |
|---|---|---|---|
| 0 | 3.0 | 4.3 | 111 |
| 1 | 3.0 | 3.4 | 113 |
| 2 | 3.0 | 2.7 | 114 |
| 3 | 3.1 | 1.7 | 116 |
| 4 | 3.2 | 0.8 | 119 |

TABLE I. Various quantities for $N_{\rm ex} = 0 - 4$ with $M_1({\rm EW})$ fixed to be 100 GeV.
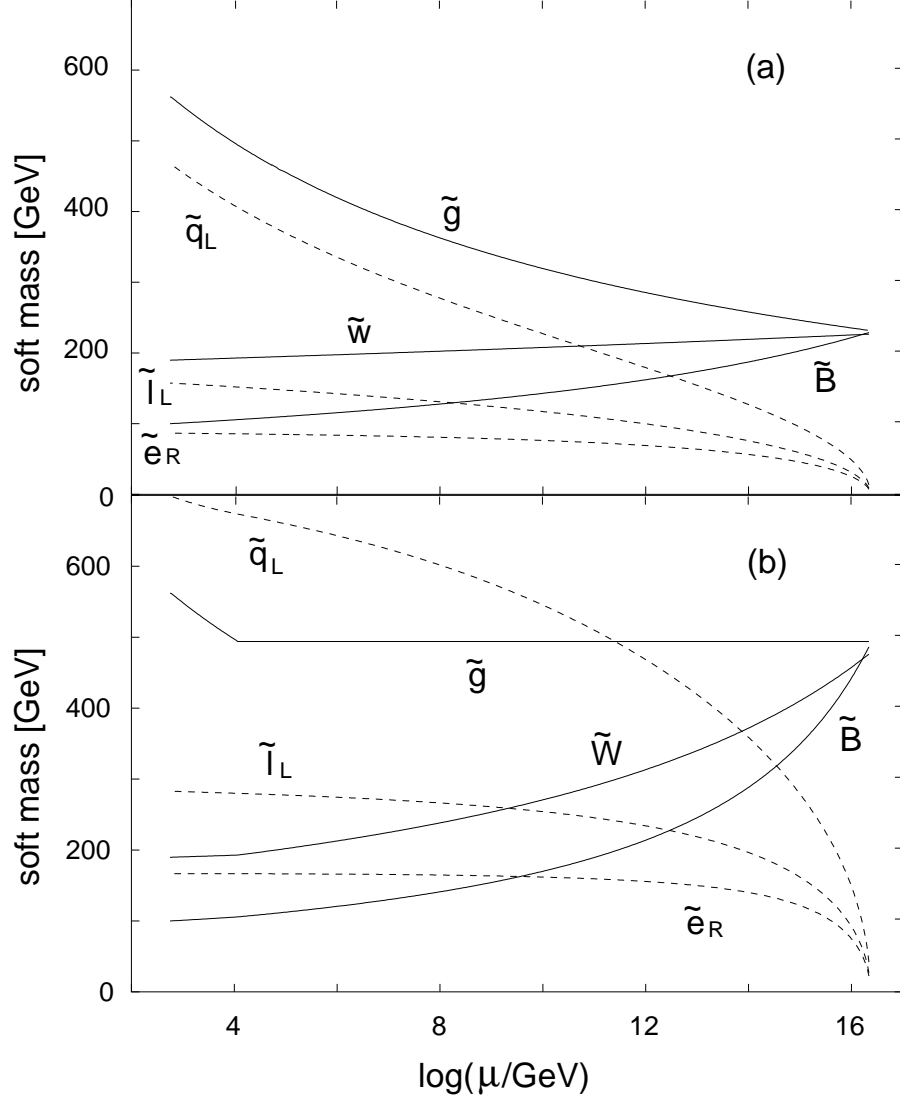
FIG. 1. RG evolutions of soft supersymmetry parameters where the B-ino mass is fixed to be 100 GeV at the electroweak scale and scalar masses are set to zero at the GUT scale. Here the number of the extra matter multiplets is $N_{\rm ex} = 0$ (pure MSSM case) for (a) and $N_{\rm ex} = 3$ above the scale $M_{\rm ex} = 10^4$ GeV for (b). The solid lines are gaugino masses and the dashed ones are scalar masses.
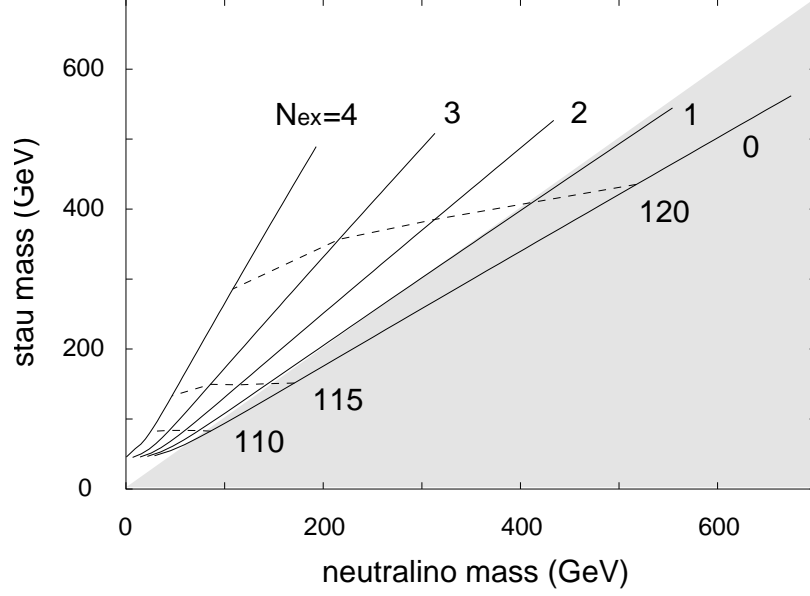
FIG. 2. The lightest neutralino mass v.s. the light stau mass at the electroweak scale in no-scale model. Each solid line corresponds to a different number of the extra matter fields, $N_{ex} = 0 - 4$ from right to left. The mass scale of the extra matters is fixed at $M_{ex} = 10^4$ GeV, and the gaugigno mass at the GUT scale is taken in the region of $M_{1/2} = 100 - 1500$ GeV. Here we take $\tan\beta = 10$. The contours of the Higgs mass are also shown in the graph(dashed). The shadow region is cosmologically disfavored, in which the stau mass is lighter than the neutralino mass.